Chapter 5

Paradoxes of Relativity

The conceptual complications of the special theory of relativity are often expressed through stories whose outcomes are counter intuitive, paradoxes. The following are the best known and provide a representative sample.

5.1 The Twin Paradox

5.1.1 The Problem

Alphonse and Gaston are twins and they are authors. Alphonse writes advertising copy and has to travel to town every day and Gaston writes novels and stays home. Each day when Alphonse is on the train going to town he is observed by Gaston. Due to their relative motion, Gaston sees Alphonse's clock running slower and thus Alphonse is aging slower than he does. At the end of the day, when Alphonse has returned home he has not aged as much as Gaston and is therefore younger. The problem is that, during the trip, Alphonse observes Gaston. He notes that Gaston's clock is the one that runs slow. He expects that, when they get back together, Gaston will be younger. When they get back together are they the same age? If there is a difference in their ages, who is younger. The clue to the problem is that Alphonse spills a drink on his shirt every day.

5.1.2 The Solution

Actually, we have already solved this paradox. This is Harry and Sally of Section 3.3.5 and 4.7. The supposed paradox here is that it seems that Alphonse and Gaston are identical. Not only are they twins but they both see the others clock run slow. The fact is that they are not identical. The clue is the answer to the paradox; Alphonse spills his drink on his shirt because he is accelerated. Gaston never spills his drink; he is not accelerated. Acceleration is knowable, velocity is not, see Section 1.2. Now that we understand that they are no longer identical, one was accerated, they can be different and it can be that one can now be older than the other one when they get back together. From Section 4.7, since the straight line time-like trajectory is the longest, the non-accelerated twin is always the oldest. The exact age difference can be computed from the trajectories of each twin in any convenient frame. The example of Harry and Sally, Section 3.3.5, is straight forward.

5.2 The Bandits and the Bullet Train

5.2.1 The Problem

The paradox is that there is a bullet train that travels at $\frac{3}{5}$c. It has a length L when measured in its rest frame which is the same as the rest frame length of a tunnel through which the train passes. Chose units of length so that $L = 1$. There are bandits living on the mountain that the tunnel passes through. Since the train carries lots of gold the bandits want to capture the train. The people on the train are aware of the probem with the bandits and post guards on the train. The bandits realize that there is a period of time when the train is entirely in the tunnel. They decide to synchronize their watchs and, just before the front of the train reaches the back of the tunnel, close both ends of the tunnel and capture the train. The train people are not worried about a capture of the train since they can put guards at both ends of the train and there is never a time when the entire train is in the tunnel. Who is correct? Can the train people use a light signal that the front guard sends to the rear to warn of the bandits when he encounters the closed tunnel?

5.2.2 The Solution

Figure 5.1 has all that you ever wanted to know about the train, the bandits, and the tunnel. It is drawn in the frame of the tunnel. Reading from the top in a clockwise fashion. The t axis and the vertical line at $x = 1$ is the back and front of the tunnel respectively. The next two lines slopped over from the vertical at $\frac{5}{3}$ are the front and back of the train. The origin, $(0, 0)$, is chosen as the event when the back of the train and the back of the tunnel coincide. The back of the train therefore passes through the origin.

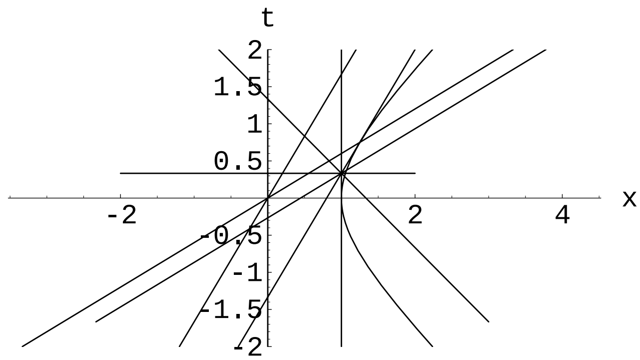

Figure 5.1: **Bullet Train, Bandits, and The Tunnel** A space-time diagram of the events associated with a bullet train that carries gold and a tunnel in which a collection of bandits have an interest.

The line representing the front of the train passes through the event ($\frac{4}{5}$, the contracted length of the train in the tunnel frame. The next curved line is the locus of events that are a proper distance of one unit from the origin. The back of the tunnel is tangent to this curve at $(1, 0)$ and the front of the train is tangent to this curve when the front of the train is at the back of the tunnel. The next two lines over from the vertical are the lines of simultaneity for the train with the two events: the back of the train is at the front of the tunnel and the front of the train is at the back of the tunnel. Note that these lines have slope $\frac{3}{5}$. From these, it is clear that the event of the front of the train at the back of the tunnel is before the event of the back of the train at the front of the tunnel to the train. This is the reverse of the order that the bandits see. They see the back of the train at the front of the tunnel before they see the front of the train at the back of the tunnel. Of course the x axis is the line of events that are simultaneous with the back of the train at the front of the tunnel to the bandits and the higher horizantal line is the set of events that are simultaneous with the arrival of the front of the train at the back of the tunnel to the the bandits. I also show the light ray from the event that is the front of the train at the back of the tunnel. This is the locus of events that is the earliest that you can know about the train hitting the closed door at the end of the tunnel. Note that the earliest that the back of the train can know about the front of the train hitting the wall is after it is about halfway in the tunnel.

The horizontal line through the event that is the front of the train at the back of the tunnel is a line of simultaneity for the bandits who are comoving with the tunnel. If the train runs exactly on time and this time is known in advance the bandits can synchronize their clocks and then the event at the intersection of this line and the events that are the front of the tunnel would be the event of their closing the gate at the front of the tunnel. The train is well inside the tunnel at this time and the capture by the bandits is complete. Had the guards sent out a radio signal at the event of the front of the train at the back of the tunnel It would arrive at the back of the train too late since the back of the train is well inside the tunnel by then. Also clearly the bandits could wait until the radio signal from the event of the front of the train at the back of the tunnel got to the front of the tunnel to close the gate. This is because, although the guards at the back of the train get the signal well before the bandits, there is no deceleration that the back of the train can initiate that will out run the radio signal to the bandits. Is seems the bandit win all around.

5.3 The Boy in the Barn

5.3.1 The Problem

A boy is a pole vault freak. He runs around a track all day to practice. He has to pass through a barn. In fact, the pole that he practices with is taken from the roof beam of the barn and is the same length as the barn when they are at rest together. He practices all day and his parents worry about him. They want to stop him and make him come in for dinner. He agrees that, if he and his pole are ever entirely in the barn, they can close the front and back doors. Since his pole is much longer than the barn, there is no problem. They will never get him. They agree to do as he says. Do they get him?

5.3.2 The Solution

This paradox is the same as the Bullet Train, Bandits, and The Tunnel in Section 5.2. The only difference here is that the moral is more positive – Parents are always right.

5.4 The Relativistic Manhole Cover

5.4.1 The Problem

5.4.2 The Solution